1. $1 + 3 + 3^2 + ... + 3^{n-1} = \frac{(3^n - 1)}{2}$

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)} = 1 + 3 + 3^2 + ... + 3^{n-1} = \frac{(3^n - 1)}{2}$

Since $s_{(1)} = 1 = \frac{(3^1 - 1)}{2} = 1$

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)} = 1 + 3 + 3^2 + ... + 3^{k-1} = \frac{(3^k - 1)}{2}$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{(3^{(k+1)} - 1)}{2}$

We observe that,

$s_{(k+1)} = 1 + 3 + 3^2 + ... + 3^{k-1} + 3^{(k+1)-1}$

$= s_{(k)} + 3^k$

$= \frac{(3^k - 1)}{2} + 3^k$

$= \frac{3^k - 1 + 6k}{2}$

$= \frac{9^k - 1}{2}$

$= \frac{3.3^k - 1}{2}$

$\therefore s_{(k+1)} = \frac{(3^{(k+1)} - 1)}{2}$

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$1 + 3 + 3^2 + ... + 3^{n-1} = \frac{(3^n - 1)}{2}$, is true for all n ∈ N.

2. $1^3 + 2^3 + 3^3 + ... + n^3 = [\frac{n(n+1)}{2}]^2$

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)} = 1^3 + 2^3 + 3^3 + ... + n^3 = [\frac{n(n+1)}{2}]^2$

Since $s_{(1)} = 1 = \frac{1(1+1)^2}{4} = 1$

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)} = 1^3 + 2^3 + 3^3 + \ldots\ldots + k^3 = [\frac{k(k+1)}{2}]^2$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{(k+1)^2[(k+1)+1]^2}{4}$

We observe that,

$$s_{(k+1)} = 1^3 + 2^3 + 3^3 + \ldots\ldots + k^3 + (k+1)^3$$

$$= s_{(k)} + (k+1)^3$$

$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$= \frac{k^2(k+1)^2+4(k+1)^3}{4}$$

$$= \frac{(k+1)^2[k^2+4(k+1)]}{4}$$

$$= \frac{(k+1)^2[k^2+4k+4]}{4}$$

$$= \frac{(k+1)^2(k+2)^2}{4}$$

$$\therefore \; s_{(k+1)} = \frac{(k+1)^2[(k+1)+1]^2}{4}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = [\frac{n(n+1)}{2}]^2$, is true for all n ∈ N.

3\. $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots\ldots + \frac{1}{1+2+3+\cdots n} = \frac{2n}{n+1}$

Solution : Let $s_{(n)}$ be the given statement ;

$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots\ldots + \frac{1}{1+2+3+\cdots n} = \frac{2n}{n+1}$$

$$s_{(n)} = 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots\ldots + \frac{2}{n(n+1)} = \frac{2n}{n+1}$$

Since $s_{(1)} = 1 = \frac{2}{1(1+1)} = 1$

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)} = 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots\ldots + \frac{2}{k(k+1)} = \frac{2k}{k+1}$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{2(k+1)}{(k+1)+1}$

We observe that,

$$s_{(k+1)} = 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots\ldots + \frac{2}{k(k+1)} + \frac{2}{(k+1)[(k+1)+1]}$$

$$= s_{(k)} + \frac{2}{(k+1)(k+2)}$$

$$= \frac{2k}{k+1} + \frac{2}{(k+1)(k+2)}$$

$$= \frac{2k(k+2)+2}{(k+1)(k+2)}$$

$$= \frac{2k^2+2k+2}{(k+1)(k+2)}$$

$$= \frac{2(k^2+k+1)}{(k+1)(k+2)}$$

$$= \frac{2(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{2(k+1)}{(k+2)}$$

$$\therefore \quad s_{(k+1)} = \frac{2(k+1)}{(k+1)+1}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots\ldots + \frac{2}{n(n+1)} = \frac{2n}{n+1}$, is true for all n ∈ N.

4. 1.2.3 + 2.3.4 + 3.4.6 + ………… + n(n+1)(n+2) = $\frac{n(n+1)(n+2)(n+3)}{4}$

Solution : Let $s_{(n)}$ be the given statement ;

The n^{th} term of the given series is n(n+1)(n+2)

$s_{(n)} = 1.2.3 + 2.3.4 + 3.4.6 + \ldots\ldots\ldots\ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

$s_{(1)} = 1.2.3 = 6 = \frac{1(1+1)(1+2)(1+3)}{4} = 6$, for n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)}$ = 1.2.3 + 2.3.4 + 3.4.6 + ……….. + k(k+1)(k+2) = $\frac{k(k+1)(k+2)(k+3)}{4}$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{(k+1)\ [(k+1)+1]\ [(k+1)+2\ ]\ [(k+1)+3]}{4}$

We observe that,

$s_{(k+1)}$ = 1.2.3 + 2.3.4 + 3.4.6 + ……….. + k(k+1)(k+2) + (k+1) [(k+1) + 1] [k+1) + 2]

$= s_{(k)}$ + (k+1) (k+2) (k+3)

$= \frac{k(k+1)(k+2)(k+3)}{4}$ + (k+1) (k+2) (k+3)

$= \frac{k(k+1)(k+2)(k+3)\ +\ 4\,[\,(k+1)(k+2)(k+3)}{4}$

$= \frac{(k+1)(k+2)(k+3)\ [\,k+4\,]}{4}$

$\therefore\ s_{(k+1)} = \frac{(k+1)\ [(k+1)+1]\ [(k+1)+2]\ [(k+1)+3]}{4}$

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

1.2.3 + 2.3.4 + 3.4.6 + ……….. + n(n+1)(n+2) = $\frac{n(n+1)(n+2)(n+3)}{4}$, is true for all n ∈ N.

5. 1.3 + 2. 3^2 + 3. 3^2 + ……. + n. 3^2 = $\frac{(2n-1)3^{n+1}+3}{4}$

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)}$ = 1.3 + 2. 3^2 + 3. 3^2 + ……. + n. 3^2 = $\frac{(2n-1)3^{n+1}+3}{4}$

Since $s_{(1)} = 3 = \frac{(2.1-1)3^{1+1}+3}{4} = \frac{12}{4} = 3$, for n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k, then

$s_{(k)}$ = 1.3 + 2. 3^2 + 3. 3^2 + ……. + n. 3^2 = $\frac{(2k-1)3^{k+1}+3}{4}$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{(2[k+1]-1)3^{(k+1)+1}+3}{4}$

We observe that,

$s_{(k+1)}$ = 1.3 + 2. 3^2 + 3. 3^2 + ... + n. 3^2 + (k+1) 3^{k+1}

$= s_{(k)} + (k+1)\, 3^{k+1}$

$= \frac{(2k-1)3^{k+1} + 3}{4} + (k+1)\, 3^{k+1}$

$= \frac{(2k-1)3^{k+1} + 3 + 4\,(k+1)3^{k+1}}{4}$

$= \frac{3^{k+1}\,(2k-1+4k+4)+3}{4}$

$= \frac{3^{k+1}\,(6k+3)+3}{4}$

$= \frac{3^{k+1}\,3(2k+1)+3}{4}$

$= \frac{(2k+1)3.3^{k+1} + 3}{4}$

$\therefore \quad s_{(k+1)} = \frac{(2[k+1]-1)3^{(k+1)+1} + 3}{4}$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

1.3 + 2. 3^2 + 3. 3^2 + ... + n. 3^2 = $\frac{(2n-1)3^{n+1}+3}{4}$, is true for all n ∈ N.

6. 1.2 + 2.3 + 3.4 + ... + n.(n+1) = $\frac{n(n+1)(n+2)}{3}$

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)}$ = 1.2 + 2.3 + 3.4 + ... + n.(n+1) = $\frac{n(n+1)(n+2)}{3}$

Since $s_{(1)} = 2 = \frac{1(1+1)(1+2)}{3} = 2$, for n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k, then

$s_{(K)}$ = 1.2 + 2.3 + 3.4 + ... + K.(K+1) = $\frac{K(K+1)(K+2)}{3}$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{(K+1)[(K+1)+1]\,[(K+1)+2}{3}$

We observe that,

$s_{(K+1)}$ = 1.2 + 2.3 + 3.4 + ... + K.(K+1) + (K+1) [(K+1) + 1]

$= s_{(K)} + (K+1)(K+2)$

$= \frac{K(K+1)(K+2)}{3} + (K+1)(K+2)$

$= \frac{K(K+1)(K+2)+ 3[(K+1)(K+2)]}{3}$

$= \frac{(K+1)(K+2)[K+3]}{3}$

$\therefore \quad s_{(k+1)} = \frac{(K+1)[(K+1) +1] [(K+1)+2}{3}$

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n $\in$ N.

1.2 + 2.3 + 3.4 + ... + n.(n+1) = $\frac{n(n+1)(n+2)}{3}$, is true for all n $\in$ N.

7. 1.3 + 3.5 + 5.7 + ... + (2n-1)(2n+1) = $\frac{n(4n^2+6n-1)}{3}$

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)}$ = 1.3 + 3.5 + 5.7 + ... + (2n-1)(2n+1) = $\frac{n(4n^2+6n-1)}{3}$

Since $s_{(1)} = 3 = \frac{1(4.1^2+61-1)}{3} = 3$, for n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k then

$s_{(k)}$ = 1.3 + 3.5 + 5.7 + ... + (2k-1)(2k+1) = $\frac{k(4k^2+6k-1)}{3}$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{(k+1)\ [4(k+1)^2+6(k+1)-1]}{3}$

We observe that,

$s_{(k)}$ = 1.3 + 3.5 + 5.7 + ... + (2k-1)(2k+1) + [2(k+1) -1] [2(k+1) + 1]

$= s_{(k)} + (2k+1)(2k+3)$

$= \frac{k(4k^2+6k-1)}{3} + (2k+1)(2k+3)$

$= \frac{k(4k^2+6k-1) +3(2k+1)(2k+3)}{3}$

$= \frac{(4k^3+6k^2-k)+(12k^2+24k+9)}{3}$

$$= \frac{4k^3+18k^2+23k+9}{3}$$

$$= \frac{(k+1)\,(4k^2+14k+9)}{3}$$

$$= \frac{(k+1)\,(4k^2+8k+6k+4+6-1)}{3}$$

$$= \frac{(k+1\,)\,[\,(4k^2+8k+4)+[(6k+6)-1]\,]}{3}$$

$$\therefore\ s_{(k+1)} = \frac{(k+1\,)\,[\,(4(k+1)^2+6(k+1)-1\,]}{3}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

1.3 + 3.5 + 5.7 + ... + (2n-1)(2n+1) = $\frac{n(4n^2+6n-1)}{3}$, is true for all n ∈ N.

8. 1.2 + 2.2^2 + $3.\,2^2$ + ... + $n.2^n$ = (n-1) 2^{n+1} +2

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)}$ = 1.2 + 2.2^2 + $3.\,2^2$ + ... + $n.2^n$ = (n-1) 2^{n+1} +2

Since $s_{(1)}$ = 2 = (1-1) 2^{1+1} +2 = 2

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k, then

$s_{(k)}$ = 1.2 + 2.2^2 + $3.\,2^2$ + ... + $k.2^k$ = (k-1) 2^{k+1} +2

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)}$ = [(k+1)-1] $2^{(k+1)+1}$ +2

We observe that,

$s_{(k+1)}$ = 1.2 + 2.2^2 + $3.\,2^2$ + ... + $k.2^k$ + $(k+1).2^{k+1}$

= $s_{(k)}$ + $(k+1).2^{k+1}$

= (k-1) 2^{k+1} +2 + $(k+1).2^{k+1}$

= 2^{k+1} [(k-1) +(k+1)] +2

= 2^{k+1}(k-1+k+1) + 2

= 2^{k+1} .2k + 2

∴ $s_{(k+1)}$ = [(k+1)-1] $2^{(k+1)+1}$ +2

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

1.2 + 2.2^2 + 3. 2^2 + ... + n.2^n = (n-1) 2^{n+1} +2 , is true for all n ∈ N.

9. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

Solution : Let $s_{(n)}$ be the given statement ;

$$s_{(n)} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

Since $s_{(1)} = \frac{1}{2} = 1 - \frac{1}{2^1} = \frac{1}{2}$

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$$s_{(k)} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = 1 - \frac{1}{2^{k+1}}$

We observe that,

$$s_{(k+1)} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots + \frac{1}{2^k} + \frac{1}{2^{k+1}}$$

$$= s_{(k)} + \frac{1}{2^{k+1}}$$

$$= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$$

$$= \frac{2^k.2^{k+1} - 2^{k+1} + 2^k}{2^k.2^{k+1}}$$

$$= \frac{2^k.2^k.2 - 2^k.2 + 2^k}{2^k.2^{k+1}}$$

$$= \frac{2^k(.2^k.2 - 2 + 1)}{2^k.2^{k+1}}$$

$$= \frac{2^k.2 - 1}{2^{k+1}}$$

$$= \frac{2^{k+1} - 1}{2^{k+1}}$$

$$\therefore \quad s_{(k+1)} = 1 - \frac{1}{2^{k+1}}$$

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$, is true for all n ∈ N.

10. $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots\ldots.. + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$

Solution : Let $s_{(n)}$ be the given statement ;

$$s_{(n)} = \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots\ldots.. + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$$

Since $\quad s_{(1)} = \frac{1}{10} = \frac{1}{(6.1+4)} = \frac{1}{10}$

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$$s_{(k)} = \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots\ldots.. + \frac{1}{(3k-1)(3k+2)} = \frac{k}{(6k+4)}$$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $\quad s_{(k+1)} = \frac{k+1}{6(k+1)+4}$

We observe that,

$$s_{(k+1)} = \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots\ldots.. + \frac{1}{(3k-1)(3k+2)} + \frac{1}{[3(k+1)-1]\,[3(k+1)+2]}$$

$$= s_{(k)} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{k}{(6k+4)} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{K(3K+2)(3K+5) + (6K+4)}{(6K+4)(3k+2)(3k+5)}$$

$$= \frac{K(3K+2)(3K+5) + 2\,(3K+2)}{(6K+4)(3k+2)(3k+5)}$$

$$= \frac{(3K+2)\,[\,k\,(3K+5) + 2\,]}{(6K+4)(3k+2)(3k+5)}$$

$$= \frac{k\,(3K+5) + 2}{(6K+4)(3k+5)}$$

$$= \frac{k^2+5k+2}{(6K+4)(3k+5)}$$

$$= \frac{k^2+3k+2k+2}{18k^2\ +12k+30k+20}$$

$$= \frac{3k(k+1)+2(k+1)}{6k(3k+2)+10(3k+2)}$$

$$= \frac{(3k+2)(k+1)}{(6k+10)(3k+2)}$$

$$= \frac{(k+1)}{(6k+10)}$$

$$\therefore \quad s_{(k+1)} = \frac{k+1}{6(k+1)+4}$$

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \text{........} + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$, is true for all n ∈ N.

11. $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \text{.......} + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

Solution : Let $s_{(n)}$ be the given statement ;

$$s_{(n)} = \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \text{.......} + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

Since $\quad s_{(1)} = \frac{1}{6} = \frac{1(1+3)}{4(1+1)(1+3)} = \frac{4}{4.2.3} = \frac{1}{6}$

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$$s_{(k)} = \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \text{.......} + \frac{1}{k(k+1)(k+2)} = \frac{k(k+3)}{4(k+1)(k+2)}$$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $\quad s_{(k+1)} = \frac{(k+1)[\,(k+1)+3]}{4[\,(k+1)+1]\,[\,(k+1)+\,2]}$

We observe that,

$$s_{(k+1)} = \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \text{.......} + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)\,[(k+1)+1]\,[(k+1)+2]}$$

$$= s_{(k)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$= \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$= \frac{k(k+3)^2 + 4}{4(k+1)(k+2)(k+3)}$$

$$= \frac{k\left(k^2 + 6k + 9\right) + 4}{4(k+1)(k+2)(k+3)}$$

$$= \frac{\left(k^3 + 6k^2 + 9k\right) + 4}{4(k+1)(k+2)(k+3)}$$

$$= \frac{k^3 + 6k^2 + 9k + 4}{4(k+1)(k+2)(k+3)}$$

$$= \frac{(k^2+5k+4)(k+1)}{4(k+1)(k+2)(k+3)}$$

$$= \frac{(k+1)(k+4)(k+1)}{4(k+1)(k+2)(k+3)}$$

$$= \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$

$$\therefore \quad s_{(k+1)} = \frac{(k+1)[(k+1)+3]}{4[(k+1)+1][(k+1)+2]}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots\ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$, is true for all n ∈ N.

12. a +a r + a r^3 + ………. + ar^{n-1} = $\frac{a\,(r^n-1)}{(r-1)}$, r ≠ 1

Solution : Let $s_{(n)}$ be the given statement ;

$$s_{(n)} = a + ar + ar^3 + \ldots\ldots + ar^{n-1} = \frac{a\,(r^n-1)}{(r-1)}$$

For n=1, $s_{(1)}$ = a = $\frac{a\,(r^1-1)}{(r-1)}$ = a

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k . then

$$s_{(k)} = a + ar + ar^3 + \ldots\ldots + ar^{k-1} = \frac{a\,(r^k-1)}{(r-1)}$$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that $s_{(k+1)} = \frac{a\,(r^{(k+1)}-1)}{(r-1)}$

We observe that ,

$$s_{(k+1)} = a + ar + ar^3 + \ldots\ldots + ar^{n-1} + a\,r^{(k+1)-1}$$

$$= s_{(k)} + a\,r^{(k+1)-1}$$

$$= \frac{a\,(r^k-1)}{(r-1)} + a\,r^k$$

$$= \frac{a\,(r^k-1)+r^k(r-1)}{(r-1)}$$

$$= \frac{a\,(r^k-1+r^{k+1}-r^k)}{(r-1)}$$

$$\therefore \quad s_{(k+1)} = \frac{a\,(r^{(k+1)}-1)}{(r-1)}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

a +a r + a r^3 + ………. +ar^{n-1} = $\frac{a\,(r^n-1)}{(r-1)}$ is true for all n ∈ N.

13. $(1+\frac{3}{1})\,(1+\frac{5}{4})\,(1+\frac{7}{9})$ ………… $(1+\frac{2\,n+1}{n^2})$ = $(n+1)^2$

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)}$ = $(1+\frac{3}{1})\,(1+\frac{5}{4})\,(1+\frac{7}{9})$ ………… $(1+\frac{2\,n+1}{n^2})$ = $(n+1)^2$

$s_{(1)}$ = 4 = $(1+1)^2$ = 4 , For n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)}$ = $(1+\frac{3}{1})\,(1+\frac{5}{4})\,(1+\frac{7}{9})$ ………… $(1+\frac{2\,k+1}{k^2})$ = $(k+1)^2$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)}$ = $[(k+1)+1\,]^2$

We observe that ,

$s_{(k+1)}$ = $(1+\frac{3}{1})\,(1+\frac{5}{4})\,(1+\frac{7}{9})$ ………… $(1+\frac{2\,k+1}{k^2})\,(1+\frac{2(\,k+1)+1}{(k+1)^2})$

= $s_{(k)}$ $(1+\frac{2\,k+3}{(k+1)^2})$

= $(k+1)^2$ $[\frac{(k+1)^2+(2k+3)}{(k+1)^2}]$

= $\frac{(k+1)^2\,(k+1)^2+(2k+3)}{(k+1)^2}$

= $(k+1)^2$ + $(2k+3)$

= k^2+ 2k + 1 +2k + 3

= k^2+ 4k + 4

= $(k+1)^2$

∴ $s_{(k+1)}$ = $[(k+1)+1\,]^2$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$(1+\frac{3}{1})\,(1+\frac{5}{4})\,(1+\frac{7}{9})$ ………… $(1+\frac{2\,n+1}{n^2})$ = $(n+1)^2$, is true for all n ∈ N.

14. $[1+\frac{1}{1}]\ [1+\frac{1}{2}]\ [1+\frac{1}{3}]\ \ldots\ldots\ [1+\frac{1}{n}] = (n+1)$

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)} = [1+\frac{1}{1}]\ [1+\frac{1}{2}]\ [1+\frac{1}{3}]\ \ldots\ldots\ [1+\frac{1}{n}] = (n+1)$

Since $s_{(1)} = [1+\frac{1}{1}] = 2 = (1+1) = 2$

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)} = [1+\frac{1}{1}]\ [1+\frac{1}{2}]\ [1+\frac{1}{3}]\ \ldots\ldots\ [1+\frac{1}{k}] = (k+1)$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = [(k+1) + 1]$

We observe that,

$s_{(k+1)} = [1+\frac{1}{1}]\ [1+\frac{1}{2}]\ [1+\frac{1}{3}]\ \ldots\ldots\ [1+\frac{1}{k}]\ [1+\frac{1}{k+1}]$

$= s_{(k)} + [1+\frac{1}{k+1}]$

$= (k+1) + [1+\frac{1}{k+1}]$

$= (k+1) + [\frac{(k+1)+1}{k+1}]$

$= \frac{[(k+1)(k+1)]+[\ (k+1)+1)]}{k+1}$

$= \frac{(k+1)\ [\ (k+1)+1]}{k+1}$

$\therefore\ s_{(k+1)} = [(k+1) + 1]$

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n $\in$ N.

$[1+\frac{1}{1}]\ [1+\frac{1}{2}]\ [1+\frac{1}{3}]\ \ldots\ldots\ [1+\frac{1}{n}] = (n+1)$, is true for all n $\in$ N.

15. $1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)} = 1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Since $s_{(1)} = 1 = \frac{1(2.1-1)(2.1+1)}{3} = 1$, for n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$$s_{(k)} = 1^2 + 3^2 + 5^2 + \ldots\ldots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{(k+1)\,[\,2(k+1)-1]\,[2(k+1)+1]}{3}$

We observe that,

$$s_{(k+1)} = 1^2 + 3^2 + 5^2 + \ldots\ldots + (2k-1)^2 + [2(k+1)-1]^2$$

$$= s_{(k)} + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1) + 3\,(2k+1)^2}{3}$$

$$= \frac{(2k+1)\ [\ k(2k-1) + 3\,(2k+1)]}{3}$$

$$= \frac{(2k+1)\ [(\,2k^2-k) + 6k+3]}{3}$$

$$= \frac{(2k+1)\ (\,2k^2-k + 6k+3)}{3}$$

$$= \frac{(2k+1)\ (\,2k^2+5k+3)}{3}$$

$$= \frac{(2k+1\,)\,(k+1)\,(2k+3)}{3}$$

$$= \frac{(k+1)\,(2k+1)\,(2k+3)}{3}$$

$$s_{(k+1)} = \frac{(k+1)\,[\,2(k+1)-1]\,[2(k+1)+1]}{3}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$, is true for all n ∈ N.

16. $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots\ldots\ldots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$

Solution : Let $s_{(n)}$ be the given statement ;

$$s_{(n)} = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \text{.........} + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$$

Since $\quad s_{(1)} = \frac{1}{4} = \frac{1}{(3.1+1)} = \frac{1}{4}$, for n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$$s_{(k)} = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \text{.........} + \frac{1}{(3k-2)(3k+1)} = \frac{k}{(3k+1)}$$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $\quad s_{(k+1)} = \frac{k+1}{3(k+1)+1}$

We observe that,

$$s_{(k+1)} = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \text{.........} + \frac{1}{(3k-2)(3k+1)} + \frac{1}{[3(k+1)-2]\,[3k+1)+1]}$$

$$= s_{(k)} + \frac{1}{(3k+1)(3k+4)}$$

$$= \frac{k}{(3k+1)} + \frac{1}{(3k+1)(3k+4)}$$

$$= \frac{k(3k+4)+1}{(3k+1)(3k+4)}$$

$$= \frac{3k^2+4k+1}{(3k+1)(3k+4)}$$

$$= \frac{(k+1)(3k+1)}{(3k+1)(3k+4)}$$

$$= \frac{k+1}{(3k+4)}$$

$$\therefore \quad s_{(k+1)} = \frac{k+1}{3(k+1)+1}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \text{.........} + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$$, is true for all n ∈ N.

17. $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \text{........} + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$

Solution : Let $s_{(n)}$ be the given statement ;

$$s_{(n)} = \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \text{........} + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

Since $\quad s_{(1)} = \frac{1}{15} = \frac{1}{3(2.1+3)} = \frac{1}{15}$, for n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$$s_{(k)} = \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots\dots + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)}$$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{k+1}{3[2(k+1)+3]}$

$$s_{(k)} = \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots\dots + \frac{1}{(2k+1)(2k+3)} + \frac{1}{[2(k+1)+1][2(k+1)+3]}$$

$$= s_{(k)} + \frac{1}{(2k+3)(2k+5)}$$

$$= \frac{k}{3(2k+3)} + \frac{1}{(2k+3)(2k+5)}$$

$$= \frac{k(2k+5) + 3}{3(2k+3)(2k+5)}$$

$$= \frac{2k^2+5k + 3}{3(2k+3)(2k+5)}$$

$$= \frac{(k+1)(2k+3)}{3(2k+3)(2k+5)}$$

$$= \frac{(k+1)}{3(2k+5)}$$

$$\therefore \; s_{(k+1)} = \frac{k+1}{3[2(k+1)+3]}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots\dots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$, is true for all n ∈ N.

18. $1 + 2 + 3 + \dots\dots + n < \frac{1}{8}(2n+1)^2$.

Solution : Let $s_{(n)}$ be the given statement ;

$$1 + 2 + 3 + \dots\dots + n < \frac{1}{8}(2n+1)^2$$

$$\frac{n(n+1)}{2} < \frac{1}{8}(2n+1)^2$$

If n=1, $\frac{1(1+1)}{2} < \frac{1}{8}(2.1+1)^2$

$$1 < \frac{9}{8}$$

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$$1 + 2 + 3 + \ldots\ldots + k < \frac{1}{8}(2k+1)^2$$

We show that the statement $s_{(n)}$ is true for n = k+1

$$1 + 2 + 3 + \ldots\ldots + k + (k+1) < \frac{1}{8}(2k+1)^2 + (k+1)$$

$$\frac{1}{8}(2k+1)^2 + (k+1)$$

$$= \frac{1}{8}[(2k+1)^2 + 8(k+1)]$$

$$= \frac{1}{8}[(4k^2 + 4k + 1) + 8k + 8]$$

$$= \frac{1}{8}(4k^2 + 12k + 9)$$

$$= \frac{1}{8}(2k+3)^2$$

$$= \frac{1}{8}[2(k+1)+1]^2$$

$$1 + 2 + 3 + \ldots\ldots + k + (k+1) < \frac{1}{8}(2k+1)^2 + (k+1)$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

19. n (n+1) (n+5) is a multiple of 3 .

Solution : Let $s_{(n)}$ be the given statement ;

n (n+1) (n+5) is a multiple of 3

if n=1, 1 (1+1) (1+5) = 9 is a multiple of 3

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

k (k+1) (k+5) is a multiple of 3

k(k+1) (k+5) = 3t (t is a natural number) ......(1)

We show that the statement $s_{(n)}$ is true for n = k+1

(k+1) [(k+1) +1] [(k+1) + 5] is a multiple of 3

(k+1) (k+2) [(k+5) + 1]

= (k+1) [k+2] (k+5) + (k+1) (k+2)

= k (k+1) (k+5) + 2(k+1) (k+5) + (k+1) (k+2)

= 3t + 2(k+1) (k+5) + (k+1) (k+2) from (1)

= 3t + (k+1) [2 (k+5) + (k+2)]

= 3t + (k+1) [2k+10 + k+2]

= 3t + (k+1) (3k+12)

= 3t + (k+1) 3(k+4)

= 3 [t + (k+1) (k+4)] { [(t+(k+1) (k+4)] is some natural number }

∴ (k+1) [(k+1) +1] [(k+1) + 5] is a multiple of 3

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

20. $10^{2n-1} + 1$ is divisible by 11 .

Solution : Let $s_{(n)}$ be the given statement ;

$10^{2n-1} + 1$ is divisible by 11

If n=1 , $10^{2.1-1} + 1 = 11$ is divisible by 11

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$10^{2k-1} + 1$ is divisible by 11

$10^{2k-1} + 1 = 11t$ (t is a natural number)

$10^{2k-1} = 11t - 1$ .... (1)

We show that the statement $s_{(n)}$ is true for n = k+1

$10^{2(k+1)-1} + 1$ is divisible by 11

$10^{2k-1} = 11t - 1$ .... from (1)

$10^{2k-1} . 10^2 = (11t - 1) 10^2$

$10^{2k-1+2} = 10^2 . 11t - 10^2$

$10^{2(k+1)-1} = 10^2 . 11t - 10^2$

Add 1 on both sides, we get

$10^{2(k+1)-1} + 1 = 10^2 . 11t - 10^2 + 1$

$10^{2(k+1)-1} + 1 = 10^2.\ 11t - (100 - 1)$

$10^{2(k+1)-1} + 1 = 100.\ 11t - 99$

$10^{2(k+1)-1} + 1 = 11 (100t - 9)$ [(100t-9) is some natural number]

$10^{2n-1} + 1$ is divisible by 11

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

21. $x^{2n} - y^{2n}$ is divisible by (x + y).

Solution : Let $s_{(n)}$ be the given statement ;

$x^{2n} - y^{2n}$ is divisible by (x + y)

If n=1 , $x^{2.1} - y^{2.1}$ is divisible by (x + y)

$x^2 - y^2$ = (x+y) (x-y) is divisible by (x + y)

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$x^{2k} - y^{2k}$ is divisible by (x + y)

$x^{2n} - y^{2n}$ = (x + y) p (p is quotient)

We show that the statement $s_{(n)}$ is true for n = k+1

$x^{2(k+1)} - y^{2(k+1)}$ is divisible by (x + y)

We know that , $x^{2k} - y^{2k}$ = (x + y) p

x^{2k} = (x + y) p - y^{2k}

x^{2k} .x = [(x + y) p - y^{2k}]x

x^{2k+1} = (x + y) px - $y^{2k}x$

$x^{2k+1} - y^{2k+1}$ = (x + y) px - $y^{2k}.x - y^{2k+1}$

$x^{2k+1} - y^{2k+1}$ = (x + y) px - y^{2k}(x+y)

$x^{2k+1} - y^{2k+1}$ = (x + y) [px - y^{2k}]

[px - y^{2k}] is a factor of (x+y)]

∴ $x^{2(k+1)} - y^{2(k+1)}$ is divisible by (x+y)

∴ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n $\in$ N.

22. $3^{2n+2} - 8n - 9$ is divisible by 8 .

Solution : Let $s_{(n)}$ be the given statement ;

$3^{2n+2} - 8n - 9$ is divisible by 8

If n=1, $3^{2.1+2} - 8.1 - 9 = 64$ is divisible by 8

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$3^{2k+2} - 8k - 9$ is divisible by 8

$3^{2k+2} - 8k - 9 = 8t$ (t is a natural number)

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $3^{2(k+1)+2} - 8(k+1) - 9$ is divisible by 8

We have that , $3^{2k+2} - 8k - 9 = 8t$

$3^{2k+2} = 8t + 8k + 9$

$3^{2k+2} . 3^2 = (8t + 8k + 9) 3^2$

$3^{2k+2} . 3^2 -8(k+1) - 9 = (72t +72k + 81) -8(k+1) - 9$

$= 72t +72k + 81 -8k-8 - 9$

$= 72t +64k +64$

$= 8(9t + 8k + 8)$

[(9t +8k+8) is some natural number]

$\therefore$ $3^{2(k+1)+2} - 8(k+1) - 9$ is divisible by 8

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n $\in$ N.

23. $41^n - 14^n$ is a multiple of 27 .

Solution : Let $s_{(n)}$ be the given statement ;

$41^n - 14^n$ is a multiple of 27

Since , $41^1 - 14^1 = 27$ is a multiple of 27

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

41^k - 14^k is a multiple of 27

41^k - 14^k = 27t , (t ∈ N)

41^k = 27t + 14^k

We show that the statement $s_{(n)}$ is true for n = k+1

We show that $41^{(k+1)}$ - $14^{(k+1)}$ is a multiple of 27

We have 41^k = 27t +14^k

41^k .41 = (27t + 14^k) 41

41^{k+1} = (27t +14^k) 41

41^{k+1}- 14^k.14 = 41. 27t + 14^k.41 - 14^k.14

41^{k+1}- 14^{k+1} = 41. 27t + 14^k (41 - 14)

41^{k+1}- 14^{k+1} = 41. 27t + 14^k.27

41^{k+1}- 14^{k+1} = 27(41t + 14^k)

[(41t + 14^k) is some natural number]

∴ $41^{(k+1)}$ - $14^{(k+1)}$ is a multiple of 27

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.