Using mathematical induction, prove each of the following statements for all $n \in N$

1. $1^2 + 2^2 + 3^2 + 4^2 + ...+ n^2 = \frac{n\,(n+1)\,(2n+1)}{6}$

Solution: Let $s_{(n)}$ be the given statement ;

$$s_{(n)} = 1^2 + 2^2 + 3^2 + 4^2 + ...+ n^2 = \frac{n\,(n+1)\,(2n+1)}{6}$$

Since $s_{(1)} = \frac{1\,(1+1)(2.1+1)}{6} = \frac{2.3}{6} = 1$, (n = 1)

The statement is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k . then

$$s_{(k)} = 1^2 + 2^2 + 3^2 + 4^2 + ...+ k^2 = \frac{k(k+1)(2k+1)}{6}$$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that $s_{(k+1)} = \frac{(k+1)\ [\,(k+1)+1\ ]\,[2(k+1)+1\ ]}{6}$

We observe that , $s_{(k+1)} = 1^2 + 2^2 + 3^2 + 4^2 + ...+ k^2 + (k+1)^2$

$$= s_{(k)} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k\,(k+1)(2k+1)+6(k+1)^2}{6}$$

$$= \frac{(k+1)\ [k(2k+1)+\ 6\,(k+1)\,]}{6}$$

$$= \frac{(k+1)\ (2k^2+k+6k+6)}{6}$$

$$= \frac{(k+1)\ (2k^2+7k+6)}{6}$$

$$= \frac{(k+1)\ (k+2)\ (2k+3)}{6}$$

$$\therefore \quad s_{(k+1)} = \frac{[(k+1)\ (k+1)+1\,]\ [2(k+1)+1]}{6}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$1^2 + 2^2 + 3^2 + 4^2 + ...+ n^2 = \frac{n\,(n+1)\,(2n+1)}{6}$, is true for all n ∈ N.

2 . 2.3 + 3.4 + 4.5 + ... + upto n terms = $\frac{n\,(\ n^2 + 6n + 1\,)}{3}$

Solution : Let $s_{(n)}$ be the given statement ;

2.3 + 3.4 + 4.5 + ………. + upto n terms = $\frac{(\ n^2 +\ 6n\ +\ 11)}{3}$

The n^{th} term of 2.3 + 3.4 + 4.5 + ………. Is (n+1) (n+2)

t^n = a+(n-1)d = 2+(n-1)1 =n+1 , 3+(n-1)1=n+2]

$s_{(n)}$ = 2.3 + 3.4 + 4.5 + ………. + (n+1) (n+2) = $\frac{n\,(\ n^2 +\ 6n + 11)}{3}$

Since $s_{(1)}$ = 2.3 = $\frac{1\,(1^2 +\ 6.1 + 11)}{3}$ = $\frac{18}{3}$ = 6 , for n =1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)}$ = 2.3 + 3.4 + 4.5 + ………. + (k+1) (k+2) = $\frac{k\,(k^2 + 6k\ +\ 11)}{3}$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)}$ = $\frac{(k+1)\ [\ (k+1)^2 + 6\,(k+1) +\ 11\ ]}{3}$

We observe that,

$s_{(k+1)}$ = 2.3 + 3.4 + 4.5 + ………. + (k+1) (k+2) + [(k+1) +1] [(k+1) +2]

= $s_{(k)}$ + (k +2) (k + 3)

= $\frac{k\,(k^2 + 6k\ +\ 11)}{3}$ + (k +2) (k + 3)

= $\frac{(k^3 + 6\,k^2 + 11\,k\,) +\ 3(\,k^2 + 5\,k + 6\,)}{3}$

= $\frac{k^3 + 9\,k^2 + 26\,k + 18}{3}$

= $\frac{(\,k+1\,)\,(\,k^2 + 8\,k\ +\ 18\,)}{3}$

= $\frac{(\,k+1\,)\,(\,k^2 + 6\,k + 2\,k\ +\ 1 + 6 + 11\,)}{3}$

= $\frac{(\,k+1\,)\ [\ (\,k^2 + 2\,k + 1\,) +\ (\,6\,k + k + 11\,)\,]}{3}$

∴ $s_{(k+1)}$ = $\frac{(\,k+1\,)\ [\ (\,k+1\,)^2 + 6\,(\,k+1\,) + 11\ ]}{3}$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

2.3 + 3.4 + 4.5 + ………. + (n+1) (n+2) = $\frac{n\,(\ n^2 +\ 6n\ +\ 1\,)}{3}$, is true for all n ∈ N.

3 . $\frac{1}{1.3}$ + $\frac{1}{3.5}$ + $\frac{1}{5.7}$ + ………. + $\frac{1}{(2n-1)(2n+1)}$ = $\frac{n}{2n+1}$

Solution : Let $s_{(n)}$ be the given statement ;

$$s_{(n)} = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots\dots\dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

Since $s_{(1)} = \frac{1}{1.3} = \frac{1}{2(1)+1} = \frac{1}{3}$, (n=1)

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k . then

$$s_{(k)} = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots\dots\dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that $s_{(k+1)} = \frac{k+1}{2(k+1)+1}$

We observe that,

$$s_{(k+1)} = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots\dots\dots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{[2(k+1)-1][2(k+1)+1]}$$

$$= s_{(k)} + \frac{1}{(2k+1)(2k+3)}$$

$$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$

$$= \frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}$$

$$= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$$

$$= \frac{k+1}{2k+3}$$

$$\therefore \quad s_{(k+1)} = \frac{k+1}{2(k+1)+1}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots\dots\dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$ is true for all n ∈ N.

4. $4^3 + 8^3 + 12^3 + \dots\dots\dots$ Upto n terms $= 16\,n^2(n+1)^2$

Solution : Let $s_{(n)}$ be the given statement ;

$$s_{(n)} = 4^3 + 8^3 + 12^3 + \dots\dots\dots + (4\,n)^3 = 16\,n^2(n+1)^2$$

$s_{(1)} = 4^3 = 16\,(1)^2(1+1)^2 = 16.4 = 64$ for n = 1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k . then

$s_{(k)}$ = 4^3 + 8^3 + 12^3 + ……… + $(4\,k)^3$ = $16\,k^2(k+1)^2$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that $s_{(k+1)}$ = $16(k+1)^2[(k+1)+1]^2$

We observe that,

$s_{(k+1)}$ = 4^3 + 8^3 + 12^3 + ……… + $(4\,k)^3$ + $[4(k+1)]^3$

= $s_{(k)}$ + $4^3(k+1)^3$

= $16\,k^2(k+1)^2$ + $4^3\,(k+1)^3$

= $16\,(k+1)^2[k^2 + 4\,(k+1)]$

= $16\,(k+1)^2\,(k^2+4k+4)$

= $16\,(k+1)^2\,(k+2)^2$

∴ $s_{(k+1)}$ = $16(k+1)^2[(k+1)+1]^2$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

4^3 + 8^3 + 12^3 + ……… + $(4\,n)^3$ = $16\,n^2(n+1)^2$ is true for all n ∈ N

5. a + (a + d) + (a +2d) + ………. Upto n terms = $\frac{n}{2}$ [2a + (n-1) d]

Solution : Let $s_{(n)}$ be the given statement ;
The n^{th} term of the given series is [a+(n-1)d]

$s_{(n)}$ = a + (a + d) + (a +2d) + ……… + [a+(n-1)d] = $\frac{n}{2}$ [2a + (n-1) d]

$s_{(1)}$ = a = $\frac{1}{2}$ [2a + (1-1) d] = a, for n =1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)}$ = a + (a + d) + (a +2d) + ……… + [a+(k-1)d] = $\frac{k}{2}$ [2a + (k-1) d]

We show that the statement $s_{(n)}$ is true for n = k+1

We show that $s_{(k+1)}$ = $\frac{k+1}{2}$ [2a + [(k+1)-1) d]]

We observe that, $s_{(k+1)}$ = a + (a + d) + (a +2d) + ………. + [a+(k-1)d] + [a + ([k+1]-1)d]

= $s_{(k)}$ + (a+kd)

= $\frac{k}{2}$ [2a + (k-1) d] + (a + k d)

= $\frac{1}{2}$ [2 k a + k^2d – k d +2 a +2 k d]

= $\frac{1}{2}$ [2 k a +2 a + k^2d + k d]

= $\frac{1}{2}$ [2 a +(k+1) + k d (k+1)]

= $\frac{1}{2}$ [(k+1) (2a+kd)]

= $\frac{k+1}{2}$ (2a+kd)

∴ $s_{(k+1)}$ = $\frac{k+1}{2}$ [2a + [(k+1)-1) d]]

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

a + (a + d) + (a +2d) + ………. + [a+(n-1)d] = $\frac{n}{2}$ [2a + (n-1) d], is true for all n ∈ N.

6. a +a r + a r^3 + ………. Upto n terms = $\frac{a\,(r^n-1)}{(r-1)}$, r ≠ 1

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)}$ = a +a r + a r^3 + ………. +ar^{n-1} = $\frac{a\,(r^n-1)}{(r-1)}$

For n=1, $s_{(1)}$ = a = $\frac{a\,(r^1-1)}{(r-1)}$ = a

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k . then

$s_{(k)}$ = a +a r + a r^3 + ………. +ar^{k-1} = $\frac{a\,(r^k-1)}{(r-1)}$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that $s_{(k+1)}$ = $\frac{a\,(r^{(k+1)}-1)}{(r-1)}$

We observe that ,

$s_{(k+1)}$ = a +a r + a r^3 + ………. +ar^{n-1} + a $r^{(k+1)-1}$

= $s_{(k)}$ + a $r^{(k+1)-1}$

$$= \frac{a\,(r^k-1)}{(r-1)} + a\,r^k$$

$$= \frac{a\,(r^k-1)+r^k(r-1)}{(r-1)}$$

$$= \frac{a\,(r^k-1+r^{k+1}-r^k)}{(r-1)}$$

$$\therefore \quad s_{(k+1)} = \frac{a\,(r^{(k+1)}-1)}{(r-1)}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$a + a\,r + a\,r^3 + \dots\dots\dots + ar^{n-1} = \frac{a\,(r^n-1)}{(r-1)}$ is true for all n ∈ N.

7. $2 + 7 + 12 + \dots\dots\dots + (5n+3) = \frac{n\,(5n-1)}{2}$

Solution : Let $s_{(n)}$ be the given statement ;

$$s_{(n)} = 2 + 7 + 12 + \dots\dots\dots + (5n+3) = \frac{n\,(5n-1)}{2}$$

$s_{(1)} = 2 = \frac{1(5.1-1)}{2} = \frac{4}{2} = 2$, For n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$$s_{(k)} = 2 + 7 + 12 + \dots\dots\dots + (5k+3) = \frac{k\,(5k-1)}{2}$$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{(k+1)[5(k+1)-1]}{2}$

We observe that,

$$s_{(k+1)} = 2 + 7 + 12 + \dots\dots\dots + (5k+3) + [5(k+1) - 3]$$

$$= s_{(k)} + (5k+2)$$

$$= \frac{k\,(5k-1)}{2} + (5k+2)$$

$$= \frac{k\,(5k-1) + 2\,(5k+2)}{2}$$

$$= \frac{5\,k^2 - k + 10\,k + 4}{2}$$

$$= \frac{5\,k^2 + 9\,k + 4}{2}$$

$= \frac{5k^2 + 5k + 4k + 4}{2}$

$= \frac{5k\,(k+1) + 4\,(k+1)}{2}$

$= \frac{(k+1)\ (5k+1)}{2}$

$\therefore\ s_{(k+1)} = \frac{(k+1)\ [5(k+1)-1]}{2}$

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

2 + 7 + 12 + ……… + (5n + 3) = $\frac{n\,(5n-1)}{2}$, is true for all n ∈ N.

8. $(1+\frac{3}{1})(1+\frac{5}{4})(1+\frac{7}{9})\ \ldots\ (1+\frac{2n+1}{n^2}) = (n+1)^2$

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)} = (1+\frac{3}{1})(1+\frac{5}{4})(1+\frac{7}{9})\ \ldots\ (1+\frac{2n+1}{n^2}) = (n+1)^2$

$s_{(1)} = 4 = (1+1)^2 = 4$, For n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)} = (1+\frac{3}{1})(1+\frac{5}{4})(1+\frac{7}{9})\ \ldots\ (1+\frac{2k+1}{k^2}) = (k+1)^2$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = [(k+1)+1\,]^2$

We observe that ,

$s_{(k+1)} = (1+\frac{3}{1})(1+\frac{5}{4})(1+\frac{7}{9})\ \ldots\ (1+\frac{2k+1}{k^2})(1+\frac{2(k+1)+1}{(k+1)^2})$

$= s_{(k)}\ (1+\frac{2k+3}{(k+1)^2})$

$= (k+1)^2\ [\frac{(k+1)^2 + (2k+3)}{(k+1)^2}]$

$= \frac{(k+1)^2\,(k+1)^2 + (2k+3)}{(k+1)^2}$

$= (k+1)^2 + (2k+3)$

$= k^2 + 2k + 1 + 2k + 3$

$= k^2 + 4k + 4$

$= (k+1)^2$

$\therefore\ s_{(k+1)} = [(k+1)+1\,]^2$

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$(1+\frac{3}{1})\ (1+\frac{5}{4})\ (1+\frac{7}{9})$ ……….. $(1+\frac{2n+1}{n^2}) = (n+1)^2$, is true for all n ∈ N.

9. $(2n+7) < (n+3)^2$

Solution : Let $s_{(n)}$ be the given statement ;

Since $(2.1+7) < (1+3)^2$ for n=1

$9 < 16$

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$(2k+7) < (k+3)^2$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $[2(k+1)+7] < [(k+1)+3]^2 = (k+4)^2$

We observe that , $2k+2+7 = 2k+7+2 < (k+3)^2 + 2$

$= k^2+6k+9 + 2$

$= k^2+6k+11 < (k^2+6k+11) + (2k+5)$

$= k^2+6k+11+2k+5$

$= k^2+8k+16$

$= (k+4)^2$

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$(2n+7) < (n+3)^2$ is true for all n ∈ N.

10. $1^2 + 2^2 + 3^2 + \dots\dots + n^2 > \frac{n^3}{3}$

Solution : Let $s_{(n)}$ be the given statement ;

$s_{(n)} = 1^2 + 2^2 + 3^2 + \dots\dots + n^2 > \frac{n^3}{3}$

Since $s_{(1)}$ = 1 $> \frac{1}{3}$ for n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)}$ = $1^2 + 2^2 + 3^2 + \dots\dots\dots + k^2 > \frac{k^3}{3}$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} > \frac{(k+1)^3}{3}$

$s_{(k+1)} = 1^2 + 2^2 + 3^2 + \dots\dots\dots + k^2 + (k+1)^2$

$= s_{(k)} + (k+1)^2$

$> \frac{k^3}{3} + (k^2 + 2k + 3)$

$= \frac{k^3+3k^2+6k+3}{3}$

$= \frac{k^3+3k^2+3k+1+3k+2}{3}$

$= \frac{(k+1)^3+(3k+2)}{3}$

$= \frac{(k+1)^3}{3} + \frac{3k+2}{3} > \frac{(k+1)^3}{3}$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$1^2 + 2^2 + 3^2 + \dots\dots\dots + n^2 > \frac{n^3}{3}$, is true for all n ∈ N.

11. 4^n - 3n – 1 is divisible by 9

Solution : Let $s_{(n)}$ be the given statement ;

4^n - 3n – 1 is divisible by 9

Since 4^1 - 3.1 – 1 = 0 is divisible by 9

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

4^k - 3k – 1 is divisible by 9

$4k$ - 3k – 1 = 9 t (t ∈ n)

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $4^{(k+1)}$ - 3(k+1) – 1 is divisible by 9

We observe that, 4^{k} - 3k – 1 = 9 t

4^{k} = 9 t +3k + 1 ......(1)

$4^{(k+1)}$ - 3(k+1) – 1

= 4^{k}.4 – 3 k – 3 - 1

= (9 t +3k + 1) 4 – 3k – 4 [4^{k} from (1)]

= 9.4 t + 12 k + 4 -3 k - 4

= 9.4 t + 9 k

= 9 (4 t + k)

∴ $4^{(k+1)}$ - 3(k+1) – 1 is divisible by 9

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

4^{n} - 3n – 1 is divisible by 9, is true for all n ∈ N.

12. $3.\,5^{2n+1} + 2^{3n+1}$ is divisible by 17

Solution : Let $s_{(n)}$ be the given statement ;

$3.\,5^{2n+1} + 2^{3n+1}$ is divisible by 17

Since $3.\,5^{2.1+1} + 2^{3.1+1} = 3.\,5^{3} + 2^{4}$ = 375 + 16 = 391

391 is divisible by 17

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$3.\,5^{2k+1} + 2^{3k+1}$ is divisible by 17

$3.\,5^{2k+1} + 2^{3k+1}$ = 17 t, (t ∈ n)

$3.\,5^{2k+1}$ = 17 t - 2^{3k+1} .... (1)

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $3.\,5^{2(k+1)+1} + 2^{3(k+1)+1}$ is divisible by 17

$3.\,5^{2(k+1)+1} + 2^{3(k+1)+1}$

= $3.\,5^{2k+1}.\,5^{2} + 2^{3k+3}$ .2

$= [\ 17t\ -\ 2^{3k+1}\ ]\ 25\ +2^{3k}.\ 2^3.2$

$= 17.25.t\ -2^{3k}.2.25\ +\ 2^{3k}.16$

$= 17.25.t\ -2^{3k}.50\ +\ 2^{3k}.16$

$= 17.25.t\ -2^{3k}[50 - 16]$

$= 17.25.t\ -2^{3k}.34$

$= 17.25.t\ -2^{3k}.2.17$

$= 17\ (25.t\ -2^{3k+1})$, $(25.t\ -2^{3k+1})$ is an integer

∴ $3.\ 5^{2(k+1)+1} +\ 2^{3(k+1)+1}$ is divisible by 17

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$3.\ 5^{2n+1} +\ 2^{3n+1}$ is divisible by 17, is true for all n ∈ N.

13. 1.2.3 + 2.3.4 + 3.4.6 + ……….. upto n terms = $\frac{n(n+1)(n+2)(n+3)}{4}$

Solution : Let $s_{(n)}$ be the given statement ;

The n^{th} term of the given series is n(n+1)(n+2)

$s_{(n)}$ = 1.2.3 + 2.3.4 + 3.4.6 + ……….. + n(n+1)(n+2) = $\frac{n(n+1)(n+2)(n+3)}{4}$

$s_{(1)}$ = 1.2.3 = 6 = $\frac{1(1+1)(1+2)(1+3)}{4}$ =6 ,   for n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)}$ = 1.2.3 + 2.3.4 + 3.4.6 + ……….. + k(k+1)(k+2) = $\frac{k(k+1)(k+2)(k+3)}{4}$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{(k+1)\ [(k+1)+1]\ [(k+1)+2\ ]\ [(k+1)+3]}{4}$

We observe that,

$s_{(k+1)}$ = 1.2.3 + 2.3.4 + 3.4.6 + ……….. + k(k+1)(k+2) + (k+1) [(k+1) + 1] [k+1) + 2]

= $s_{(k)}$ + (k+1) (k+2) (k+3)

= $\frac{k(k+1)(k+2)(k+3)}{4}$ + (k+1) (k+2) (k+3)

= $\frac{k(k+1)(k+2)(k+3)\ +\ 4\ [\ (k+1)(k+2)(k+3)]}{4}$

$$= \frac{(k+1)(k+2)(k+3)\ [k+4]}{4}$$

$$\therefore\ s_{(k+1)} = \frac{(k+1)\ [(k+1)+1]\ [(k+1)+2]\ [(k+1)+3]}{4}$$

$\therefore$ The formula is true for n =k+1

$\therefore$ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

1.2.3 + 2.3.4 + 3.4.6 + ………… + n(n+1)(n+2) = $\frac{n(n+1)(n+2)(n+3)}{4}$, is true for all n ∈ N.

14. $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots\dots\dots\dots + \text{upto n terms} = \frac{n}{24}(2n^2+9n+13)$

Solution : Let $s_{(n)}$ be the given statement ;

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots\dots\dots\dots + \frac{1^3+2^3+3^3+\cdots n^3}{1+3+5+\cdots 2n-1} = \frac{n}{24}(2n^2+9n+13)$$

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots\dots\dots + \frac{n^2(n+1)^2}{4\,n^2} = \frac{n}{24}(2n^2+9n+13)$$

$$s_{(n)} = \frac{n^2(n+1)^2}{4\,n^2} = \frac{(n+1)^2}{4} = \frac{n}{24}(2n^2+9n+13)$$

Since $s_{(1)} = 1 = \frac{1}{24}(2.1^2+9.1+13) = \frac{24}{24} = 1$, for n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots\dots\dots\dots + \frac{1^3+2^3+3^3+\cdots k^3}{1+3+5+\cdots 2k-1} = \frac{k}{24}(2k^2+9k+13)$$

$$s_{(k)} = \frac{(k+1)^2}{4} = \frac{k}{24}(2k^2+9k+13)$$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{k+1}{24}\ [2(k+1)^2 + 9\,(k+1) + 13]$

We observe that,

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots\dots\dots + \frac{(k+1)^2}{4} + \frac{[\,(k+1)+1]^2}{4}$$

$$= s_{(k)} + \frac{(k+2)^2}{4}$$

$$= \frac{k}{24}(2k^2+9k+13) + \frac{(k+2)^2}{4}$$

$$= \frac{k}{24}\quad (2k^2+9k+13) + 6(k^2+4k+4)$$

$$= \frac{1}{24}\ [\ (2k^3+9k^2+13k) + (6k^2+24k+24)\ ]$$

$= \frac{1}{24}\ (2k^3+9k^2+13k + 6k^2+24k+24)$

$= \frac{1}{24}\ (2k^3+15k^2+37k+24)$

$= \frac{(k+1)}{24}\ (2k^2+13k+24)$

$= \frac{(k+1)}{24}\ (2k^2+4k+9k+2+9+13)$

$= \frac{(k+1)}{24}\ (2k^2+4k+2) + (9k+9)+13)$

$\therefore \quad s_{(k+1)} = \frac{k+1}{24}\ [2(k+1)^2 + 9\ (k+1) + 13]$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots\ldots\ldots\ldots + \frac{1^3+2^3+3^3+\cdots n^3}{1+3+5+\cdots 2n-1} = \frac{n}{24}(2n^2+9n+13)$, is true for all n ∈ N.

15. $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots\ldots$ upto n terms $= \frac{n\,(n+1)^2\,(n+2)}{12}$

Solution : Let $s_{(n)}$ be the given statement ;

$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots\ldots + (1^2 + 2^2 + 3^2 + \ldots. + n^2) = \frac{n\,(n+1)^2\,(n+2)}{12}$

The n^{th} term of the given series is $\frac{n\,(n+1)(2n+1)}{6}$

$s_{(n)} = 1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots\ldots + \frac{n\,(n+1)(2n+1)}{6} = \frac{n\,(n+1)^2\,(n+2)}{12}$

Since $s_{(1)} = 1 = \frac{1\,(1+1)^2\,(1+2)}{12} = \frac{12}{12} = 1$, for n=1

The statement $s_{(n)}$ is true for n = 1

Assume that the statement $s_{(n)}$ is true for n = k , then

$s_{(k)} = 1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots\ldots + \frac{k\,(k+1)(2k+1)}{6} = \frac{k\,(k+1)^2\,(k+2)}{12}$

We show that the statement $s_{(n)}$ is true for n = k+1

We show that, $s_{(k+1)} = \frac{(k+1)\ [(k+1)+1]^2\ [(k+1)+2]}{12}$

We observe that,

$s_{(k)} = 1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots\ldots + \frac{k\,(k+1)(2k+1)}{6} + \frac{(k+1)\ [(k+1)+1]\ [2(k+1)+1]}{6}$

$= s_{(k)} + \frac{(k+1)\,(k+2)(2k+3)}{6}$

$$= \frac{k\,(k+1)^2\,(k+2)}{12} + \frac{(k+1)\,(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+2)\ \{k(k+1) + 2(2k+3)]}{12}$$

$$= \frac{(k+1)(k+2)(\,k^2+k+4k+6)}{12}$$

$$= \frac{(k+1)(k+2)(\,k^2+5k+6)}{12}$$

$$= \frac{(k+1)(k+2)\,[(k+2)(k+3)]}{12}$$

$$= \frac{(k+1)\,(k+2)^2\,(k+3)}{12}$$

$$\therefore \quad s_{(k+1)} = \frac{(k+1)\ [(k+1)+1]^2\ [\,(k+1)+2]}{12}$$

∴ The formula is true for n =k+1

∴ By the principle of mathematical induction $s_{(n)}$ is true for all n ∈ N.

$1^2 \ \ (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots\ldots + \frac{n\,(n+1)(2n+1)}{6} = \frac{n\,(n+1)^2\,(n+2)}{12}$, is true for all n ∈ N.