T.S. Intermediate Board Mathematics IA Model Paper

(Extracted from tsbie.cgg.gov.in)

Mathematics Paper - I(A)

Time : 3 Hours *Max. Marks : 75*

SECTION A

I. Very Short Answer Type Questions : **10 × 2 = 20**

(i) Answer **ALL** questions.

(ii) Each question carries **TWO** marks.

1. If $f : R \to R$ is defined by $f(x) = \frac{1-x^2}{1+x^2}$, then show that :

$$f(\tan \vartheta) = \cos 2\vartheta .$$

2. Find the domain of the real valued function :

$$f(x) = \frac{1}{(x^2-1)(x+3)} .$$

3. If

$$\begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix},$$

then find the values of x, y, z and a.

4. Define Rank of a matrix.

5. If the vectors $-3\vec{\imath} + 4\vec{\jmath} + \lambda\vec{k}$ and $\mu\vec{\imath} + 8\vec{\jmath} + 6\vec{k}$ are collinear vectors, then find λ and μ.

6. Find the vector equation of the plane passing through the points (0, 0, 0), (0, 5, 0) and (2, 0, 1).

7. Find the angle between the planes $\bar{r} \cdot (2\vec{i} - \vec{j} + 2\bar{k}) = 3$ and $\bar{r} \cdot (3\vec{i} + 6\vec{j} + \bar{k}) = 4$.

8. Find the cosine function whose period is 7.

9. What is the value of $\tan 20° + \tan 40° + \sqrt{3} \tan 20° \tan 40°$?

10. For any $x \in R$, prove that $\cosh^4 x - \sinh^4 x = \cosh(2x)$.

SECTION B

II. Short Answer Type Questions : **5 × 4 = 20**

(i) Answer **ANY FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then find A^4.

12. If the points whose position vectors are $3\vec{i} - 2\vec{j} - \bar{k}$, $2\vec{i} + 3\vec{j} - 4\bar{k}$, $-\vec{i} + \vec{j} + 2\bar{k}$ and $4\vec{i} + 5\vec{j} + \lambda\bar{k}$ are coplanar, then show that :

$$\lambda = \frac{-146}{17}.$$

13. If $|\bar{a}| = 13$, $|\bar{b}| = 5$ and $\bar{a} \cdot \bar{b} = 60$, then find $|\bar{a} \times \bar{b}|$.

14. Prove that :

$$\cos\frac{2\pi}{7} \cos\frac{4\pi}{7} \cos\frac{8\pi}{7} = \frac{1}{8}.$$

15. Solve the equation :

$$\sqrt{3} \sin\vartheta - \cos\vartheta = \sqrt{2}.$$

16. Prove that :

$$\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} = \frac{\pi}{2}.$$

17. In Δ ABC, show that :

$$b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2} = s.$$

SECTION C

III. Long Answer Type Questions : ***5 × 7 = 35***

*(i) Answer **ANY FIVE** questions.*

*(ii) Each question carries **SEVEN** marks.*

18. If f : A → B, g : B → C be bijections, then show that :

$$(gof)^{-1} = f^{-1}og^{-1}.$$

19. Using mathematical induction, prove the statement :

$$a + ar + ar^2 + \ldots\ldots \text{ upto } n \text{ terms} = \frac{a(r^n - 1)}{(r-1)}, \quad r \neq 1.$$

20. If

$$\begin{bmatrix} a & a^2 & 1 + a^3 \\ b & b^2 & 1 + b^3 \\ c & c^2 & 1 + c^3 \end{bmatrix} = 0 \text{ and } \begin{bmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{bmatrix} \neq 0,$$

then show that :

$$abc = -1.$$

21. Solve the system of equations

$$x + y + z = 1,\ 2x + 2y + 3z = 6,\ x + 4y + 9z = 3$$

by using Cramer's rule.

22. Find the volume of the tetrahedron whose vertices are (1, 2, 1), (3, 2, 5), (2, –1, 0) and (–1, 0, 1).

23. If A, B, C are angles in a triangle, then prove that :

$$\sin A + \sin B + \sin C = 4 \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}.$$

24. If r : R : r_1 = 2 : 5 : 12, then prove that the triangle is right angled at A.